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Code No. : 22111 SP

**VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD**  
**B.E. II Year II-Semester Special Examination, September-2017**

**Mathematics - IV**

Time: 3 hours

Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

**Part-A (10 × 2 = 20 Marks)**

1. Find the Laplace transform of  $f(t) = e^{-t} \sinh t$ .
2. Find  $\left\{L \frac{\sin t}{t}\right\}$ .
3. State and prove linearity property of Fourier transforms.
4. Obtain the Fourier sine transform of  $f(x) = \begin{cases} 1, & 0 \leq x < 2 \\ 0, & x > 2 \end{cases}$
5. Find the Z-transform of the sequence  $\left\{\frac{1}{n!}\right\}$ .
6. If  $Z\{f_n\} = \frac{z^2 - 3z + 5}{(z-1)(z+2)}$ , find  $\lim_{n \rightarrow \infty} f_n$ .
7. Show that  $f(z) = \bar{z}$  is not differentiable at  $z = 0$ .
8. Evaluate  $\oint_c \frac{e^{z^2}}{z-2} dz$ , where  $c: |z|=1$ .
9. Expand  $f(z) = \sin z$  in powers of  $\left(z - \frac{\pi}{4}\right)$ .
10. Find the fixed points of the bilinear transformation  $w = \frac{z-1}{z+1}$ .

**Part-B (5 × 10 = 50 Marks)**  
**(All bits carry equal marks)**

11. a) Find  $L^{-1}\left\{\ln\left(1 + \frac{1}{s^2}\right)\right\}$ .  
b) Find  $L\left\{\int_0^t u e^{-u} \sin u du\right\}$ .
12. a) Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  and hence evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$ .  
b) Find the Fourier cosine transform of  $f(x) = e^{-x^2}$ .
13. a) Determine the Z-transform of  $\{2^{-n} + (n-1)^2\}$ .  
b) State convolution theorem of Z-transform and hence find  $Z^{-1}\left\{\frac{z^2}{(z-1)(z-2)}\right\}$ .
14. a) Construct the analytic function whose real part is  $x^3 - 2xy - 3xy^2$  using Milne-Thomson method.  
b) Apply Cauchy's integral formula to evaluate  $\oint_c \frac{e^z}{z(1-z)^2} dz$ , where  $c: |z|=2$ .

15. a) Find the Laurent's series expansion of  $f(z) = \frac{1}{(z+1)(z+3)}$  valid for

(i)  $1 < |z| < 3$  and (ii)  $|z| > 3$ .

b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$ .

16. a) Solve  $y'' + y = 2e^t, y(0) = 0, y'(0) = 2$  using Laplace transforms.

b) Show that  $F_s \{x f(x)\} = -\frac{d}{ds} F_c(s)$  and hence find the Fourier sine transform of  $x e^{-x}$ .

17. Answer any **two** of the following:

a) If  $Z\{f_n\} = \frac{z^2}{z^2 + 1}$ , find  $f_0, f_1$  and  $f_2$ .

b) Obtain Cauchy-Riemann equations in polar form.

c) Determine the bilinear transformation that maps the points  $0, -i, -1$  in the  $z$ -plane to the points  $i, 1, 0$  in the  $w$ -plane respectively.

